647510 **BSCmaMJ202x-1C** **Seat No :_____**

B.Sc. Semester - 1 (NEP-2020) Examination

OCT/NOV-2024

MATH: Calculus and Solid Geometry (Major-2)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) **Answer any one.** **(05)**

(1) State and prove Cauchy's mean value theorem.

(2) Assuming the validity of expansion, expand $\log(1+sinx)$ up to first three terms, where $-\frac{\pi}{2} < x < \frac{3\pi}{2}$, will expansion exists for $x = -\frac{\pi}{2}$.

Que.1(B) **Answer any one.** **(05)**

(1) Derive power series expansion of $f(x) = cosx$ in terms of x.

(2) For > 0, show that $\frac{x}{1+x^2} < tan^{-1}x < x$.

Que.2(A) **Answer any one.** **(05)**

(1) State and prove L'Hospital's rule for $\frac{\infty}{\infty}$ form.

(2) Evaluate: (i) $\lim_{x\to 0} \frac{log(1+x)-x+\frac{1}{2}x^2}{x^3}$ (ii) $\lim_{x\to\frac{\pi}{2}}(secx - tanx)$

Que.2(B) **Answer any one.** **(05)**

(1) If $\lim_{x\to\frac{\pi}{2}} \frac{cos^2x}{a-bcosecx} = 1$ then find the value of a and b.

(2) Evaluate: (i) $\lim_{x\to 0} \frac{sinx-xcosx}{sinx-x}$ (ii) $\lim_{x\to\frac{\pi}{2}}(sinx)^{tanx}$

Que.3(A) **Answer any one.** **(05)**

(1) Define: Sphere. Find the equation of the sphere through the four points $(0,0,0), (-a,b,c), (a,-b,c), (a,b,-c)$.

(2) Get the equation of tangent plane to the sphere with $A(2,7,6)$ and $B(3,-4,5)$ as extremities of a diameter. Also find the equation of normal to this sphere at the given point A.

Que.3(B) **Answer any one.** **(05)**

(1) Find the equation of two tangent planes to the sphere $x^2+y^2+z^2 = 9$ which passes through the line $x+y = 6, x-2z = 3$.

(2) Show that the plane $2x-2y+z+12 = 0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z = 3$ and find the point of contact.

Que.4(A) **Answer any one.** **(05)**

(1) Find equation of right circular cylinder of which guiding curve is a circle $x^2+y^2+z^2 = 4, x+y+z = 3$.

(2) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve $ax^2+2hxy+by^2+2gx+2fy+c = 0; z = 0$.

Que.4(B) **Answer any one.** **(05)**

(1) Find equation of cylinder whose generator is parallel to X-axis and touching the sphere having center $(1,2,0)$ and radius 3.

(2) Find equation of right circular cylinder whose radius is 2 and axis is $\frac{x-1}{2} = \frac{y}{3} = z-3$.

Que.5(A) **Answer any one.** **(05)**

(1) State: Lagrange's theorem and verify it for the function

$$f(x) = (x-1)(x-2)(x-3), \forall x \in [0,4].$$

(2) Evaluate: (i) $\lim\limits_{X \to \frac{\pi}{2}} \frac{log\left(x-\frac{\pi}{2}\right)}{tanx}$ (ii) $\lim\limits_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{sin^2 x}\right)$

Que.5(B) **Answer any one.** **(05)**

(1) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity and find the equation of the sphere which has this circle for one of the great circle.

(2) Find the equation of right circular cylinder with axis $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r.

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